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The Lubrication of Rollers

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THE LUBRICATION OF ROLLERS

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CONTENTS

When lubricated rollers are run together they are separated by a hydrodynamically formed oil film. The thickness of this film has been measured by a capacitance method up to loads of 1000 Lb. per inch of face ($1·76 \times 10^8$ dyn cm^{-1}) for conditions of pure rolling and for conditions of rolling with sliding such as exist at the contacts of gear teeth. It has been found, at low loads, that the film thickness varies inversely with load and proportionately with speed, as simple hydrodynamic theory suggests, but that the actual thickness is approximately one-half of the theoretical thickness. At higher loads, of greater practical importance, the film thickness is shown to be of the order $1\,\mu$ ($\sim 4 \times 10^{-5}$ in.) which is greater than that predicted by simple theory. Experimental evidence is presented that this failure of the simple theory is due to the increased viscosity of the oil under pressure and to the deformation of the surfaces by the load. The film thickness, at practical loads, has been found to vary little with load, less than proportionately with speed but to vary greatly with the temperatures of the surfaces.

Estimates of the temperature reached by the oil in its passage through the conjunction of the surfaces have been made from measurements of the electrical resistivity of the oil. This temperature, which depends upon load and the peripheral speeds and which may exceed 250° C, appears to have little influence upon the film thickness. From this it is argued that the film thickness is largely determined by the conditions, on the entry side of the conjunction, ahead of the region in which the viscous losses and heating of the oil become intense.

1. Introduction

The nature of the lubrication of gears and roller bearings is still a subject of speculation. The nominal line contacts made by their loaded surfaces distinguish these machine elements from other elements and present the problem whether such surfaces can be separated by a hydrodynamic film. The doubt arises because the concentration of the load over a narrow band implies that the passage of oil between the surfaces must be opposed by pressures of extreme intensity. Recently, however, the electrical resistance between two disks running in line contact has been measured and values up to a few megohms have been found (Crook 1957). Such high values are clearly incompatible with solid contact and support the conclusion that despite the concentration of the load, the surfaces can be separated by a hydrodynamic film.

A pioneering theoretical discussion of the problem was published by Martin (1916). He adapted Reynolds's hydrodynamic theory (1886) of Tower's experiments on journal bearings (1885) to the kinematic conditions of disks such as those of figure 1 *a* and showed, by repeated integration of Reynolds's differential equation, that the relation between F, the total load per unit face width and h_{0D}, the minimum film thickness (figure 2) is given by

$$F = 2{\cdot}448\ \eta r\ (u_1 + u_2)/2h_{0D}, \tag{1·1}$$

where η is the viscosity of the oil, r the radius of the disks and u_1, u_2 their peripheral speeds. At practical loads this equation gives film thicknesses (h_{0D}) which are small compared with the roughnesses of machined surfaces. For example, it gives a film thickness of only $0{\cdot}24\,\mu$ ($9{\cdot}5 \times 10^{-6}$ in.) in a typical practical case ($F = 1{\cdot}76 \times 10^8$ dyn cm^{-1}, 1000 Lb. in.$^{-1}$; $\eta = 0{\cdot}5$ P; $r = 3{\cdot}81$ cm, 1·5 in.; $u_1 = u_2 = 914$ cm s^{-1}, 30 ft. s^{-1}).

In the derivation of equation (1·1), Martin assumed that the oil viscosity was constant and that the disks retained their initial cylindrical form. Later it will be demonstrated experimentally that these assumptions have some validity at low loads, but at practical loads their inadequacy is evident from the pressures Martin's theory predicts. These pressures, which are obtained at the penultimate stage of integration in the derivation of equation (1·1), are given by the full-line pressure curve of figure 2, for the practical load already cited. The dashed pressure curve gives the Hertzian pressure distribution (Hertz 1896) for dry disks at the same load. A comparison of these curves shows that the disks would be deformed by the predicted hydrodynamic pressures to an extent even greater than by the Hertzian pressures, which by symmetry must flatten the disks over their band of contact. It can also be seen that Martin's theory gives a maximum pressure of $1{\cdot}1 \times 10^{10}$ dyn cm^{-2} (approximately 1×10^4 atm). At that pressure the viscosity of the oil would be enormously increased (Bridgman 1949). Because the hydrodynamic pressures depend greatly upon the shape of the gap through which the oil passes it is necessary to take into account the distortion of the disks, and because of the high pressures, the variation of viscosity with pressure should also be considered. Moreover, as a result of the heating due to the shearing of the oil, the temperature of the oil will be raised as the oil passes through the gap and in a complete theory the change of viscosity due to this temperature rise should be considered as well.

The problem presented by this multiplicity of effects has been discussed theoretically by Grubin (1949) and Poritsky (1952), but it appears to be too complex to admit of a definite analytical solution. Furthermore, there is no body of unequivocal experimental results to satisfy the obvious practical need to know the order of magnitude of the film thickness nor to guide the theoretical discussion. It is probable that the further development of a solution depends upon the collection of such results.

Attempts to measure the thickness of the oil film have been based upon the discharge voltage of the film (Cameron 1954) and upon the capacitance between rolling disks (Lewicki 1955). The discharge voltage measurements lead to a film thickness of the order of 1×10^{-3} in. ($25\,\mu$). This result is improbable because, if the surfaces were so widely separated, surfaces with a roughness of less than 1×10^{-4} in. would not become smoother in running as they are known to do, nor would the resistance between disks finished to

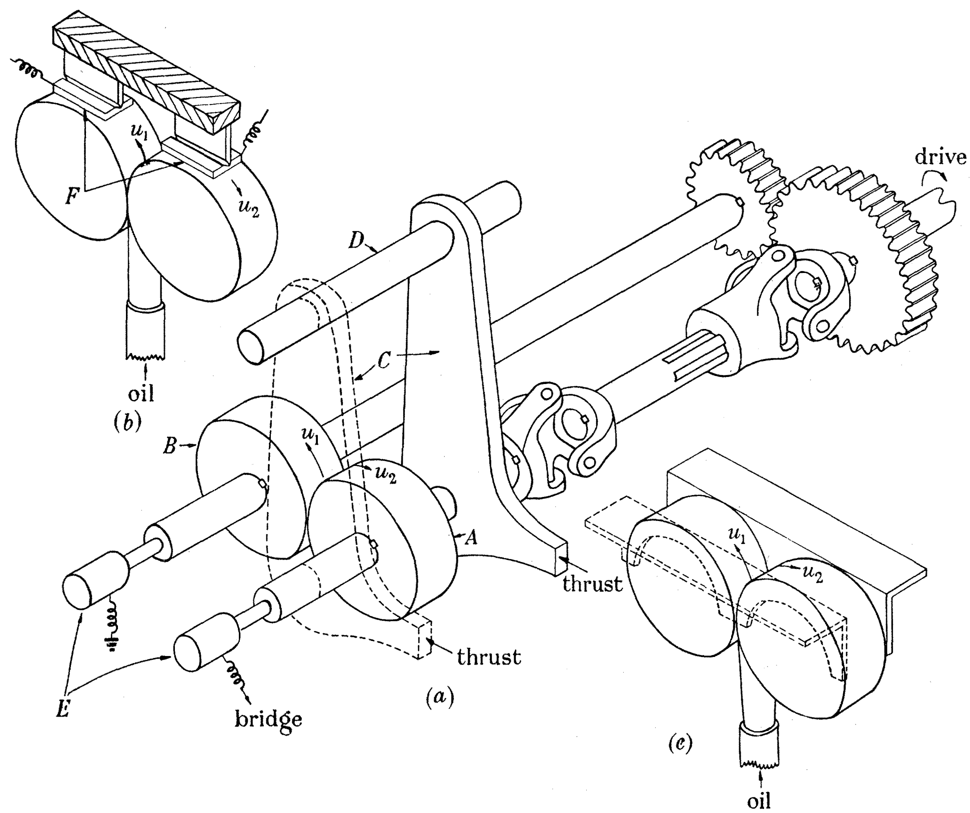

FIGURE 1. The disk machine. (*a*) *A* and *B*, the disks; *C*, the swinging arms; *D*, axle held in the machine frame (the machine frame is not shown); *E*, the mercury contacts. (*b*) *F*, the pads; (*c*) the side shields.

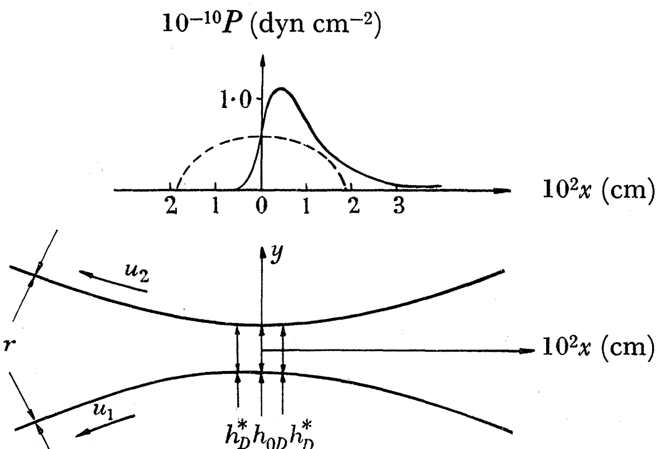

FIGURE 2. Pressure distributions. —, Martin; – – –, Hertz. (The lower figure indicates the location of the pressures.)

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a similar degree be less than 1 ohm at the start of running and rise to a megohm in an uninterrupted run of sufficient length (Crook 1957). A weakness of the method lies in the attribution to the oil in the gap of an electrical behaviour observed under the vastly different physical conditions of static calibration experiments. The capacitance method does not suffer so severely from a similar weakness for it rests upon the dielectric constant of the oil. It has been shown (Bridgman 1949), up to pressures of the same magnitude as those within the gap, that the variation with pressure of the dielectric constant of some non-polar organic liquids is consistent within a few per cent, with the increase in the number of dipoles per unit volume arising from the increase in density. It may reasonably be assumed that a non-polar oil will behave similarly. The maximum change in its dielectric constant would then be no more than 20%. The major defect of the method is that, because the capacitance between the disks depends upon the shape of the gap, a knowledge of the distortion of the disks must be presupposed before the capacitance results can be interpreted. However, the method can safely be used at very low loads where the distortions are negligible.

This paper describes results obtained by measuring the capacitance between the disks, and results obtained by a new method which is independent of the distortions of the disks and which is, therefore, still valid at high loads. The oil passing between the disks leaves their conjunction by forming films on both disk surfaces (Crook 1957). In the new method stationary unloaded pads ride upon these surface films and the capacitance between each pad and its disk is measured. From these capacitances the rate of oil flow between the disks is deduced and, by a relationship of general validity, the thickness of the oil film which separates the disks at the position of the pressure maximum can be calculated. Because the pads are unloaded, and therefore, because no high pressures can exist in the oil beneath them, there is no distortion to be considered in interpreting the pad capacitances, nor is there any uncertainty about the dielectric constant of the oil. By using measurements of disk capacitance and the new method in combination, it has been possible to find the loads at which the effects of the pressure increase of viscosity and the distortion of the disks become appreciable. It has also been possible with the new method to measure film thicknesses well into the range of practical loads under conditions of rolling, and also under conditions of rolling with sliding such as exist at the contacts of gear teeth.

2. Experimental

2·1. *The disk machine*

It is imperative that no variation in the thickness of the oil film across the faces of the disks, i.e. in the axial direction, should be imposed by a mal-alinement of the disks. This requirement is met in the apparatus (figure 1*a*) by making the disk A self-alining in the horizontal plane. This freedom of alinement is obtained by running the shaft of the disk A in spherical races in the arms C, each of which hangs freely from the axle D carried by the machine frame. (The machine frame is not shown.) The disk B runs in bearings in the machine frame and therefore rotates on a fixed axis. The disks are loaded together by applying equal thrusts to both arms C as indicated in the figure. The thrusts are imposed by a simple system which is not shown. The shafts of both disks are driven by the gears on the right of the figure and the freedom of alinement of the disk A is preserved by the use of a Cardan shaft to couple this disk to its gear. In principle the machine is of a type described by Merritt (1935).

The disk *A* was isolated electrically so that the disk capacitance could be measured. It was essential that the large stray capacitance due to the insulation should remain constant. This capacitance was independent of variations in the ambient temperature to within $\pm 1\,\mu\mu$F when insulation of P.T.F.E. was used. The leakage resistance was greater than $2 \times 10^8\,\Omega$ at 1 kV. Electrical connexion to each disk was made through the mercury contacts *E* which were of a type described by Kenyon (1954).

The disks were 3 in. (7·62 cm) in diameter and 0·75 in. (1·91 cm) thick and were made from fully hardened 0·8 % carbon steel. They were ground and lapped. The generators of their cylindrical surfaces were straight to within 5×10^{-6} in. ($0{\cdot}13\,\mu$) and their peak-to-peak surface roughness was less than 2×10^{-6} in. ($0{\cdot}05\,\mu$). When assembled in the machine the eccentricity of the disks was less than 5×10^{-5} in.

2·2. *The pads*

The arrangement of the pads is shown in figure 1*b*. It is obviously essential that only the oil passing between the disks should reach the pads and the Perspex side shields indicated in figure 1*c* were used to exclude extraneous oil. The pads were hardened steel plates 0·5 in. (1·27 cm) wide and 1·5 in (3·81 cm) long and each was lapped on one face to a flatness of 5×10^{-6} in. ($0{\cdot}13\,\mu$) and a peak-to-peak surface finish of 2×10^{-6} in. ($0{\cdot}05\,\mu$). A knife-edge arrangement (figure 1*b*) was used to hold the pads yet leave them self-alining. The load on each pad was less than one pound.

2·3. *The oil*

A mineral oil was used (turbine oil to Admiralty specification OM100) at an inlet temperature of 50° C which was thermostatically maintained. The viscosity of the oil is given as a function of temperature in figure 3.

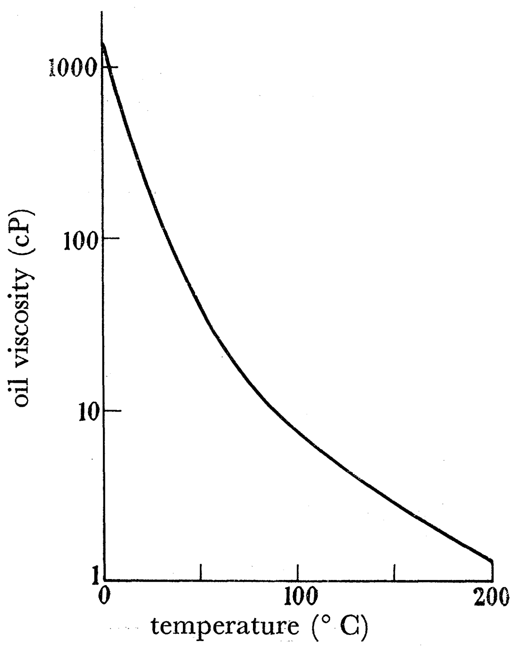

FIGURE 3. The variation of oil viscosity with temperature.

The dielectric constant of the oil at room temperature and atmospheric pressure as measured at 1 kc/s was found to be 2·30. Evidence that the oil is non-polar was provided by refractive index measurements in the visible spectrum which showed the electron component of the dielectric constant to be 2·19; 95 % of the total. It is to be expected, therefore, that the dielectric constant of this oil should vary only on account of changes in density; i.e. the dielectric constant ϵ should satisfy the Clausius–Mosotti relation

$$(\epsilon-1)/(\epsilon+2)d=\text{const.}, \tag{2·3·1}$$

where d is the density. Measurements of ϵ and d at temperatures up to 250° C satisfied equation (2·3·1).

2·4. *Measurement of capacitance*

The capacitances were measured at a frequency of 1 kc/s and at a peak potential of 0·2 V by a Wien bridge (Hague 1945). This also gave the resistances of the oil films, since the bridge has to be balanced with respect to both resistive and reactive components. The output from the bridge, as presented by a c.r.o., consisted of a sinusoidal trace interrupted by a few sharp and random peaks. The origin of these peaks was not isolated; probably some were due to adventitious matter in the oil. However, these peaks were disregarded and in making measurements attention was concentrated upon reducing to zero the amplitude of the sinusoidal trace.

Both with disk and pad measurements the film capacitance was taken as the capacitance of the machine when appropriately connected and running, minus the machine capacitance when stationary with the conducting surfaces separated by P.T.F.E. 0·030 in. (0·076 cm) thick and the remaining crevices oil-filled, plus the calculated capacitance between the surfaces when separated in that manner.

3. Measurements of disk capacitance

3·1. *Results*

The measured capacitances between disks rolling at a peripheral speed of 19·6 ft. s^{-1} (597 cm s^{-1}) and under loads up to 1000 Lb. per inch of face ($1·76\times10^{8}$ dyn cm^{-1}) are given in figure 4*b*. The experimental points lie close to a smooth curve and it is evident that the random error in the measurement of capacitance was only a few $\mu\mu$F.

3·2. *Discussion*

For the conditions of the above experiments, equation (1·1) becomes, after rearrangement,

$$1/h_{0D}=1·79\times10^{-4}\,F/\eta, \tag{3·2·1}$$

where h_{0D} is in cm when η is in P and F in dyn cm^{-1}.

When it is assumed that the disks are undeformed, that ϵ has its normal value and both the entry and recess sides of the conjunction of the disks are completely oil-filled, the disk capacitance, C_D, is given by

$$C_D=2·38\,h_{0D}^{-\frac{1}{2}}, \tag{3·2·2}$$

where C_D is in $\mu\mu$F when h_{0D} is in cm. (It has been assumed in the derivation of equation (3·2·2) that the field lines between the conducting surfaces are straight. This is justified because the mutual inclinations of the surfaces are small over the region of close approach which contributes most to the capacitance.)

These equations together give the linear relation between C_D^2 and the load shown by the dashed line in figure 4*a*. The oil viscosity was taken as 0·4 P; its value at the inlet temperature. Now, contrary to the assumption which has been made, the recess side of the disks is not completely oil-filled, and, therefore, it is to be expected that the actual values of C_D^2 should be less than those given by the dashed line. However, the experimental results

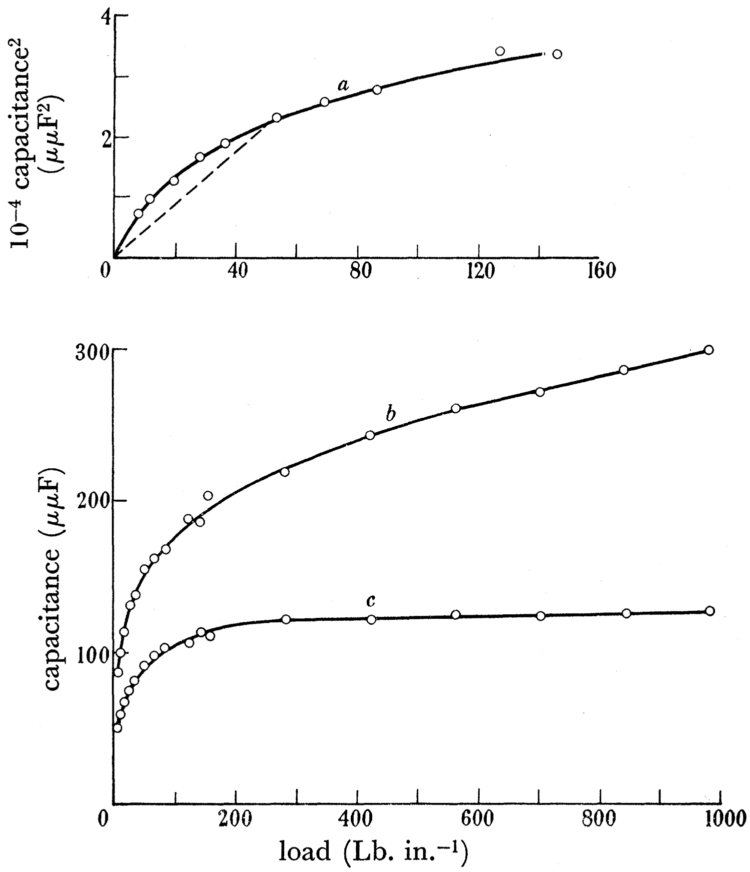

FIGURE 4. Disk and pad capacitances as functions of load. (Disks rolling at a peripheral speed of 19·6 ft. s^{-1} (597 cm s^{-1}).) (*a*) Disk capacitance squared; —○— experimental; – – –, calculated. (*b*) Disk capacitance. (*c*) Pad capacitance.

(figure 4*a*) show, at low loads where the assumptions of Martin's theory are most likely to be true, that the values of C_D^2 are greater than those given by the dashed line. Because of the inverse relation between C_D^2 and h_{0D}, this requires the film thicknesses to be less than those predicted by equation (3·2·1) and, therefore, by equation (1·1).

At high loads the disk capacitance for a given film thickness must be enhanced by the deformation of the disks. Nevertheless, at these loads the experimental values of C_D^2 are less than those given by the dashed line. This implies at high loads that the film thicknesses are greater than those predicted by Martin's theory and in fact, the almost constant capacitance observed between the pads and their disks at loads above 200 Lb. in^{-1}. (figure 4*c*) shows that there the film thickness is almost independent of load.

4. Measurement of film thickness

4·1. *Experimental*

In figure 5 simultaneous measurements of disk and pad capacitances, such as those of figures 4*b* and *c*, are replotted; the pad capacitance being given as a function of the disk capacitance. The pad capacitance is proportional to the disk capacitance up to values which correspond to a load of approximately 100 Lb. in.$^{-1}$, but beyond that the disk capacitance outstrips the pad capacitance. The pad capacitance depends only upon the flow of oil between the disks and, therefore, only upon the thickness of the oil film between them. So long as the disks are undeformed the disk capacitance will similarly depend

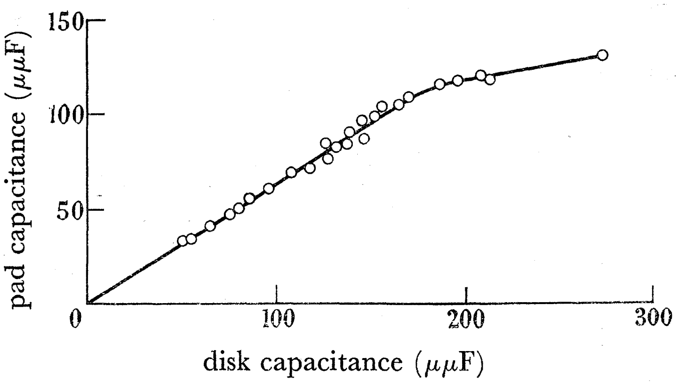

Figure 5. Pad capacitance as a function of disk capacitance.

Table 1. The relationship of pad capacitance to disk capacitance

C_D = disk capacitance; C_F = capacitance between pad and disk on fixed axis shaft; C_S = capacitance between pad and disk on self-alining shaft.

conditions of running	C_F/C_D	C_S/C_D	limit of proportionality C_D (μμF)
(*a*) both shafts 1500 rev/min, viscosity of inlet oil 0·4 P, temperature of inlet oil 50° C	0·63	0·62	180
(*b*) both shafts 375 rev/min, viscosity of inlet oil 0·4 P, temperature of inlet oil 50° C	0·65	0·67	230
(*c*) both shafts 1500 rev/min, viscosity of inlet oil 0·74 P, temperature of inlet oil 34° C	0·61	0·59	—
(*d*) fixed axis shaft 2300 rev/min, self alining shaft 1500 rev/min, viscosity of inlet oil 0·4 P, temperature of inlet oil 50° C	0·62	0·66	180

upon this film thickness and, therefore, the ratio of pad to disk capacitance should be independent of film thickness. The proportional part of figure 5 shows this to be true when the thickness is varied by load and the effectively constant value of the ratio given by the results *a*, *b* and *c* summarized in table 1 shows that it is still true when the film thickness is also varied by changing speed and oil viscosity.

At high loads the disk capacitance must be enhanced by the deformation. Consequently, the bend of the pad capacitance, disk capacitance curve (figure 5) will be taken as marking the onset of the deformation of the disks.

Evidence supporting this interpretation is provided by the fact that the bend, or limit of proportionality, occurs at a smaller capacitance and, therefore, at a greater film thickness as the speed is increased (cf. (*a*) and (*b*) of table 1). According to simple hydrodynamic theory the maximum film pressure is proportional to speed and inversely proportional to the three-halves power of the film thickness. It may be assumed, as a first approximation, that deformation becomes appreciable at a certain maximum pressure. It is then evident that simple hydrodynamic theory requires the film thickness at the limit of proportionality to increase with speed. There is, therefore, qualitative agreement between the experiments and the hydrodynamic theory.

The experiments suggest that the disks are undeformed up to the limit of proportionality. It will be assumed in this range that the pad capacitance C_F is related to the disk capacitance C_D by

$$C_F/C_D = 0\cdot63, \tag{4·1·1}$$

where 0·63 is the mean of the ratios in table 1.

The result (*d*) of table 1 shows that even when the speeds of the disks are unequal both pads obey equation (4·1·1) with reasonable accuracy.

4·2. *Theoretical*

If C_D can be expressed in terms of the film thickness when the disks are undeformed, equation (4·1·1) provides the connexion by which C_F can be related to the film thickness. This latter relation will be true whether the disks be undeformed or not since C_F is dependent only upon the film thickness or, more precisely, only upon the flow of oil between the disks.

It is necessary to consider first which film thickness between the disks determines the flow. This thickness may be found by examining the fundamental Navier–Stokes hydrodynamic equations which, after appropriate simplification (Reynolds 1886; Martin 1916; Gatcombe 1945), become in the co-ordinates of figure 2

$$\eta \partial^2 u/\partial y^2 = \partial P/\partial x, \tag{4·2·1}$$

where u is the velocity of the oil in the positive x direction and P is the pressure. It follows from this equation that $\partial u/\partial y$ is constant at the pressure maximum because, there $\partial P/\partial x$ is zero. (Equation (4·2·1) still applies when, because of the influence of temperature and pressure, η is a function of x. Although in the derivation an incompressible medium is assumed, the equation is also true at the pressure maximum for a compressible oil. This is because, in the disk problem, the essential feature of an incompressible medium is that $\partial d/\partial x$ should be zero; a condition which a compressible oil also satisfies at the pressure maximum. However, the equation does rest upon the assumption that η is independent of y and if this be not so the equation should be replaced by

$$\frac{\partial \eta}{\partial y}\frac{\partial u}{\partial y} + \eta \frac{\partial^2 u}{\partial y^2} = \frac{\partial P}{\partial x}.$$

Under rolling conditions $\partial u/\partial y$ is zero, where $\partial^2 u/\partial y^2$ is zero and, consequently, a variation of η with y, even if it should occur, does not affect the null value of $\partial^2 u/\partial y^2$ at the pressure maximum. In the presence of sliding this is not so and it is more likely that there will be a temperature difference between the disks producing a variation of η with y. The place where $\partial P/\partial x$ is zero is then no longer truly coincident with the place where $\partial u/\partial y$ is constant and it is most probable that the latter place is shifted towards the entry side.) Therefore, if h^*_{DP} be the separation of the disks at the pressure maximum the rate of volume flow of compressed oil, Q_P, through unit face width of the gap is given by

$$Q_P = \tfrac{1}{2}(u_1 + u_2) h^*_{DP}. \tag{4·2·2}$$

The pads, however, detect the rate of volume flow of uncompressed oil Q and, by analogy, an apparent film thickness h^*_D can be defined so that

$$Q = \tfrac{1}{2}(u_1 + u_2) h^*_D. \tag{4·2·3}$$

Thus, the flow Q, and therefore the pad capacitance also, can be related at all loads to h^*_D. However, at the low loads for which C_D of equation (4·1·1) is to be evaluated the compression of the oil is insignificant and h^*_D can be regarded as the true film thickness at the pressure maximum. All that now remains is to express the disk capacitance, C_D, of undeformed disks in terms of h^*_D.

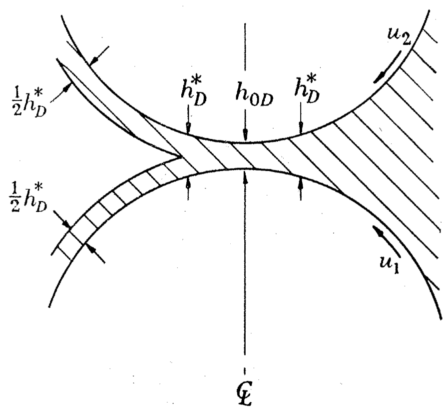

Figure 6. The disposition, assumed in calculating the disk capacitance, of the oil between the disks.

This expression for C_D depends upon the ratio of h^*_D to the minimum film thickness, h_{0D} (figure 2) and also upon the extent of the oil-filled region on the recess side. The ratio

$$h^*_D / h_{0D} = 1·23 \tag{4·2·4}$$

will be chosen (Martin 1916; McEwen 1952) because in Martin's theory this value makes $\partial P/\partial x$ zero where the pressure falls to zero on the recess side, and thereby satisfies an essential condition for continuity of flow. Although it has been shown (§ 3·2) that the film thickness given by Martin's theory is too great at low loads it will be assumed that the reason for that does not radically affect equation (4·2·4).

If $\partial P/\partial x$ is zero where the pressure falls to zero on the recess side, the film thickness there must also be h^*_D. Consequently, it will be assumed that the oil between the undeformed disks is disposed as shown in figure 6. After h^*_D has been substituted for h_{0D} by equation (4·2·4) and after taking the disk dimensions into account, the disc capacitance is then given by

$$C_D = 2·25 h^{*-\frac{1}{2}}_D, \tag{4·2·5}$$

where C_D is in $\mu\mu$F when h^*_D is in cm. Equation (4·2·5) provides, for undeformed disks, the required relation between the disk capacitance and film thickness.

4·3. *Conclusion*

Together, equations (4·1·1) and (4·2·5) give

$$C_F = 1{\cdot}42 h_D^{*-\frac{1}{2}}, \tag{4·3·1}$$

where C_F is in $\mu\mu$F when h_D^* is in cm. This relates the pad capacitance to the apparent film thickness at the pressure maximum. The true film thickness there, h_{DP}^*, may be derived from h_D^* if the compressibility of the oil and maximum pressure be known.

Although equation (4·3·1) is a result of considering undeformed disks it must be emphasized that, for the purpose of calibrating the pads, the undeformed disks simply provide a way of producing films of measured thickness. The pad capacitances are obviously determined by the films upon which the pads ride and are obviously independent of whether these films issue from undeformed or deformed disks. Consequently, equation (4·3·1) is valid at all loads.

The above calibration has been presented here because it is thought to be the most accurate. It rests, however, upon equation (4·2·5) which is based upon the assumption that the disks are undeformed and upon the assumed disposition of the oil between them. An alternative calibration which does not require such assumptions has been found. Because of the uncertainty which exists over the magnitude of the film thickness it is particularly important that the calibration should be checked and for that reason the alternative method, which is entirely independent, is given in the appendix.

The alternative method gives 1·67 in place of the numerical factor in equation (4·3·1) and would, therefore, give film thicknesses approximately 40 % greater than that equation. This difference is small in relation to the uncertainty which has existed in the order of magnitude of the film thickness and is without effect upon the relative values of h_D^* which can therefore be known to an accuracy (~5 %) limited only by the accuracy of the capacitance measurements.

5. Film thickness measurements

5·1. *Results with rolling disks*

In these experiments the disks were driven at equal rotational speeds by gears of one-to-one ratio. The inlet temperature of the oil was 50° C.

In figure 7 *a* the reciprocal of the film thickness (h_D^*) is plotted for loads up to 100 Lb. in.$^{-1}$ ($1{\cdot}76\times10^7$ dyn cm^{-1}). The values of h_D^* were obtained from measurements of pad capacitance by equation (4·3·1). Film thickness measurements at a load of 17·2 Lb. in.$^{-1}$ ($3{\cdot}02\times10^6$ dyn cm^{-1}) are plotted against speed in figure 7 *b*.

Film thickness results at higher loads are given in figure 8 for three speeds of rolling. The oil film thickness increases with speed and slowly decreases with load. This decrease is less marked at the lower speed where the film thickness is small. Double logarithmic plots of apparent film thickness against peripheral speed are given for two loads in figure 9; the plots are linear.

In an experiment at a constant load of 700 Lb.in.$^{-1}$ ($1{\cdot}23\times10^8$ dyn cm^{-1}) the oil supply to the disks was interrupted for about 25 min. The apparent film thickness and also the temperatures recorded by a stationary thermocouple resting against the moving

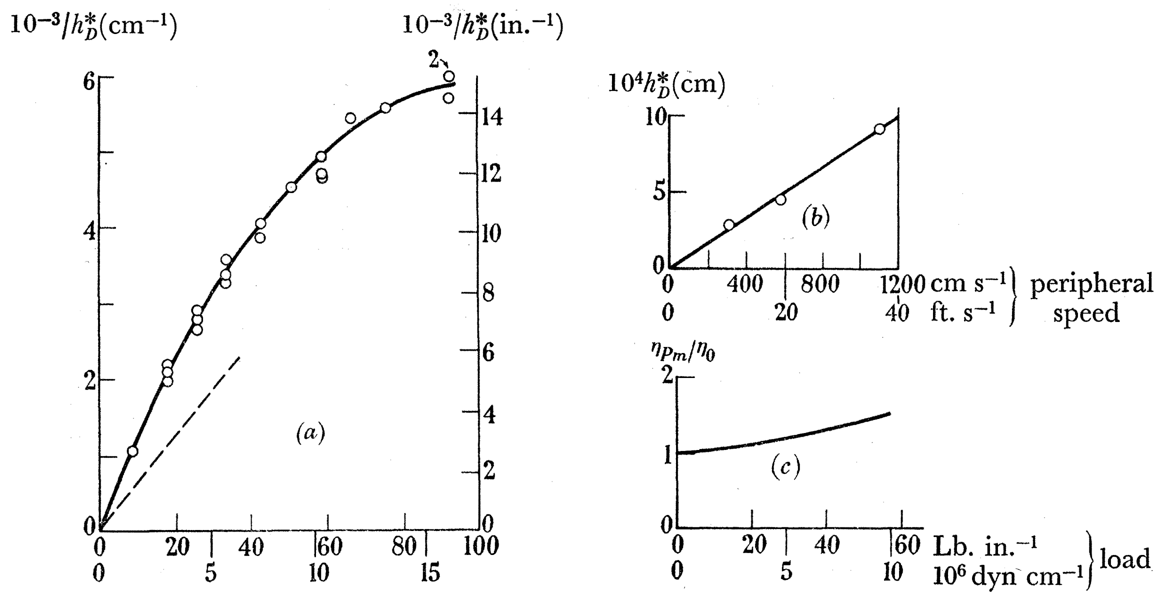

FIGURE 7. Results at low loads. (*a*) The reciprocal of film thickness as a function of load: —○—, experimental; – – –, calculated. (Disks rolling at a peripheral speed of 19·6 ft. s^{-1} (597 cm s^{-1}).) (*b*) Film thickness as a function of rolling speed (experimental). (Load 17·2 Lb. in.$^{-1}$ (3·02 × 10^6 dyn cm^{-1}).) (*c*) The relative increase of viscosity with load (calculated).

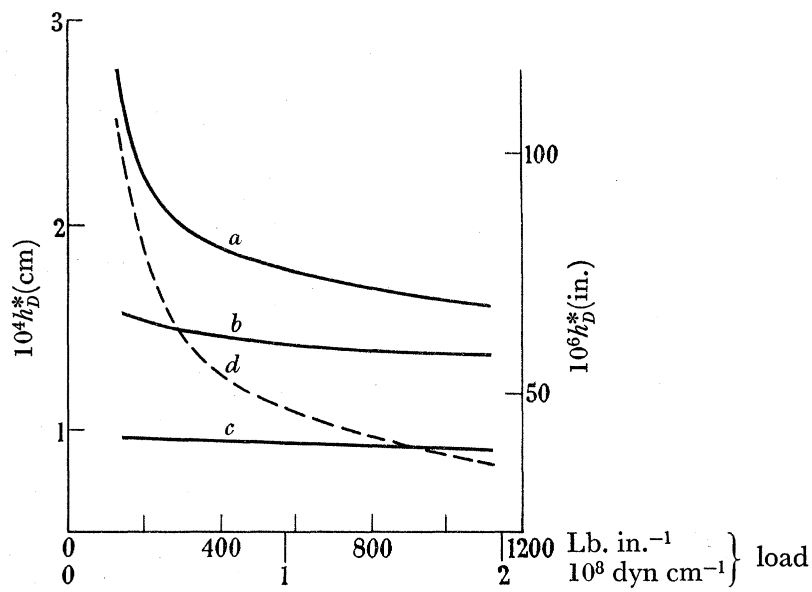

FIGURE 8. The variation of film thickness with load. —, Rolling disks. Peripheral speeds: (*a*) 38·3 ft. s^{-1} (1170 cm s^{-1}); (*b*) 19·6 ft. s^{-1} (597 cm s^{-1}); (*c*) 6·6 ft. s^{-1} (201 cm s^{-1}). – – –, With sliding. (*d*) Peripheral speeds 19·6 ft. s^{-1} (597 cm s^{-1}) and 30·2 ft. s^{-1} (920 cm s^{-1}).

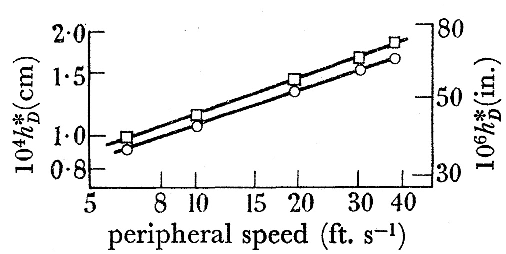

FIGURE 9. The film thickness between rolling disks as a function of speed. ○, Load 1120 Lb. in.$^{-1}$ (1·97 × 10^8 dyn cm^{-1}); □, load 400 Lb. in.$^{-1}$ (7·02 × 10^7 dyn cm^{-1}).

surface of one disk were measured and are plotted in figure 10. Over the period of no oil supply the apparent film thickness decreased from $1{\cdot}4\,\mu$ ($5{\cdot}5\times10^{-5}$ in.) to $1{\cdot}1\,\mu$ ($4{\cdot}3\times10^{-5}$ in.), whilst the thermocouple temperature rose from 55 to 65° C. When the oil supply was resumed there was a time delay of approximately 4 min before the film thickness and temperature again reached their initial values.

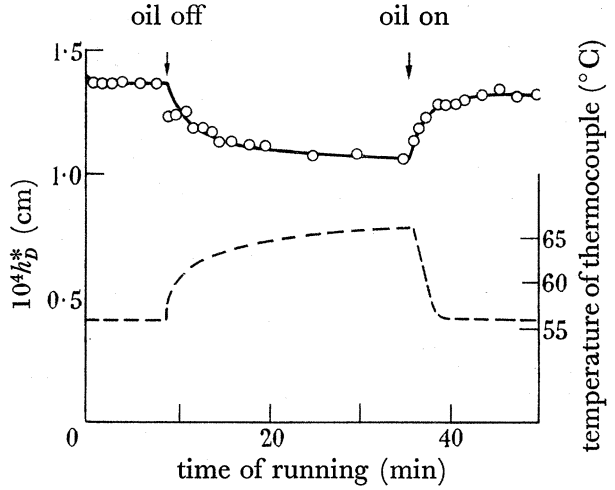

FIGURE 10. The variations of film thickness and disk temperature with time after interrupting the oil supply to rolling disks. (Peripheral speed 19·6 ft. s^{-1} (597 cm s^{-1}), load 700 Lb. in.$^{-1}$ ($1{\cdot}23\times10^{8}$ dyn cm^{-1}). —○—, Film thickness; – – –, temperature.

5·2. *Discussion*

The experimental results of figure 7 confirm equation (1·1) at low loads in two respects. Graph (*a*) of the figure shows that the film thickness is inversely proportional to the load up to approximately 15 Lb. in.$^{-1}$ ($2{\cdot}63\times10^{6}$ dyn cm^{-1}), whilst graph (*b*) shows that the film thickness is proportional to speed.

The dashed line in figure 7*a* gives the variation of h_D^{*-1} with load to be expected from equation (1·1). The viscosity was taken as the viscosity of the inlet oil (0·4 P). In agreement with a conclusion from the consideration of disk capacitances (§3·2) the figure shows the film thicknesses at low loads to be less than those predicted by equation (1·1). This agreement is not independent so long as the interpretations of pad and disk capacitances depend upon the same assumptions. However, the interpretation of pad capacitance given in the appendix rests upon different assumptions and if the calibration equation of the appendix had been applied to the measured pad capacitances the deduced film thicknesses would still have been less than those predicted by equation (1·1). This provides independent evidence that the film thicknesses at low loads are less than those given by Martin's theory.

The discrepancy between the experimental and theoretical thicknesses could be resolved by assuming that the effective viscosity of the oil is less than the viscosity of the inlet oil. An effective viscosity of approximately 0·2 P would be required. Such a diminution of viscosity might result from the frictional heating. Cameron (1952) has given an expression for the coefficient of friction of rolling disks from which it may be shown that the energy dissipated per unit flow of oil, H/Q is given by

$$H/Q = 2{\cdot}4F\,(rh_D^*)^{-\frac{1}{2}}. \qquad (5{\cdot}2{\cdot}1)$$

Because of the discrepancy between the film thicknesses, equation (5·2·1) is not exactly applicable but it will suffice for the estimation of viscosities. At zero load h_D^* is infinite and it is clear from equation (5·2·1) that there can be no frictional heating. After the insertion of the experimental values of h_D^* equation (5·2·1) gives $3{\cdot}3\times10^7$ erg cm^{-3} for H/Q at a load of 1×10^6 dyn cm^{-1} and $3{\cdot}4\times10^8$ erg cm^{-3} at a load of 5×10^6 dyn cm^{-1}. If it be assumed that no heat is lost to the disks' surfaces the consequent temperature rises of the oil as it passes between the disks are 1·7 and 17° C, respectively. By the viscosity temperature curve of the oil (figure 3) the outlet viscosities of the oil would be 0·9 and 0·5, respectively of the inlet viscosity. Thus, according to the hypothesis the outlet viscosity of the oil should diminish rapidly with load, whereas the decreasing rate of change of film thickness with increase of load shown by figure 7*a* requires the effective viscosity to increase with load. It is concluded, therefore, that an apparently low effective viscosity of the oil over the initial load range cannot be attributed to frictional heating and furthermore, that the frictional heat does not remain in the oil as was assumed, but that the majority of the heat passes into the disks' surfaces. The explanation of the discrepancy between the film thicknesses may indeed not lie in the viscosity but elsewhere in equation (1·1).

The pressure dependence of viscosity can be expressed in the form (Bradbury, Mark & Kleinschmidt 1951)

$$\eta_P = \eta_0 \exp \delta P, \tag{5·2·2}$$

where η_P is the viscosity at the pressure P. In curve *c* of figure 7 the ratio of the viscosity at the pressure maximum to η_0 is plotted as a function of load; η_{P_m} was calculated from equation (5·2·2) at the pressure maximum, P_m, given by (Martin 1916)

$$P_m = 0{\cdot}7F\,(rh_D^*)^{-\frac{1}{2}}, \tag{5·2·3}$$

and the experimental values of h_D^* were used. Because of the thickness anomaly, equation (5·2·3) is not exact but it will indicate the magnitude of the viscosity changes to be expected. The constant δ of equation (5·2·2) was taken as $1{\cdot}6\times10^{-9}$ dyn^{-1} cm^2, $1{\cdot}1\times10^{-4}$ Lb.$^{-1}$ in.^{2} (Bradbury *et al.* 1951).

The increase in viscosity shown by curve *c* of figure 7 is approximately consistent with the non-linearity of curve *a*. It is reasonable, therefore, to interpret the departure from linearity of the latter curve, which commences at about 15 Lb. in.$^{-1}$, as being due to the pressure dependence of viscosity.

The change of film thickness with speed at constant load shown in figure 7*b* will be accompanied by changes in P_m and, therefore, in η_{P_m}. However, it can be shown from equations (5·2·2) and (5·2·3) that the variation in η_{P_m} to be expected is no more than 4%. The observed proportionality of h_D^* with speed is, therefore, not inconsistent with the suggestion that the departure of curve *a* from linearity is due to the pressure effect.

In the above discussion of the results at low loads, the pressures developed in the oil films were too low to compress the oil significantly and it was assumed that h_D^* and h_{DP}^* (see equations (4·2·2) and (4·2·3)) were equal. At high loads this is no longer true. It will be assumed from a load of 280 Lb. in^{-1} ($4{\cdot}92\times10^7$ dyn cm^{-1}) to the highest load used, 1120 Lb. in.$^{-1}$ ($1{\cdot}97\times10^8$ dyn cm^{-1}) that the maximum oil pressure is approximately equal to the maximum Hertzian pressure. The maximum Hertzian pressure, P_H, between the disks is given in Lb. in.$^{-2}$ by (Hertz 1896)

$$P_H = 2640\sqrt{F}, \tag{5·2·4}$$

where F is expressed in Lb. in.$^{-1}$. When F is 280 Lb. in.$^{-1}$, P_H is 44000 Lb. in.$^{-2}$ and when F is 1120 Lb. in.$^{-1}$, P_H is 88000 Lb. in.$^{-2}$. The densities of oil at high pressures, which will be denoted here by d_P, have been measured experimentally (Bradbury *et al.* 1951). Precise values of the ratio d_0/d_P depend upon the temperature; at a pressure of 45000 Lb. in.$^{-2}$, d_0/d_P is 0·88 at 98° C and is 0·83 at 220° C. The ratios at a pressure of 90000 Lb. in.$^{-2}$ are 0·84 and 0·78, respectively. The compression of the oil film due to pressure will be mitigated by the expansion of the oil as it becomes heated in its passage between the disks. It is probable, therefore, that the ratio h^*_{DP}/h^*_D is greater than 0·8. The true value of the film thickness at the pressure maximum, h^*_{DP}, therefore, lies between h^*_D and $0·8h^*_D$ at loads up to 1120 Lb. in.$^{-1}$. It should be noted, however, that the variation of h^*_{DP}/h^*_D from a load of 280 Lb. in.$^{-1}$ to a load of 1120 Lb. in.$^{-1}$ will be less than from unity to 0·8. Consequently, the relative values of film thickness will not be greatly affected by compression. In conclusion, it can be said that it is not imperative to correct h^*_D for compression, particularly in relation to relative values, before proceeding with the discussion of film thicknesses at high loads.

The double logarithmic plot of film thickness against speed of figure 9 shows that at high loads the film thickness is no longer proportional to speed but obeys a relation of the form

$$h^*_D \propto u^{\frac{1}{3}},$$

whilst the curves of figure 8 show, with the exception of the initial part of curve *a*, that the film thickness is almost independent of load. This behaviour is at variance with Martin's theory and is due to the deformation of the disks and the increase of viscosity with pressure.

The rapidly varying initial part of curve *a* is interpreted as a transition region before the effect of elastic deformation is fully established. It can be seen from equation (5·2·3) that the load at which a critical value of P_m develops increases with film thickness. If the onset of a significant amount of elastic distortion be associated with a critical value of P_m this provides an explanation of why the initial curvatures of graphs *a*, *b* and *c* decrease in that order.

The experiments in which the oil supply was interrupted (figure 10) were prompted by a deduction from resistance measurements (Crook 1957) that the oil forming the films upon the disks is retained by the disks for many cycles and is only slowly replaced by fresh oil from the jet. It was in fact suggested that the oil from the jet does not readily merge with the oil films already carried by the disks. If this be true, the oil films on the disks should continue to provide effective lubrication after the oil jet has been stopped. The results (figure 10) show that the films did continue to provide effective lubrication, for when the oil jet was stopped the film thickness fell from 1·4 μ only to 1·1 μ after 25 min running. Moreover, the temperature rise at the surface of the disk indicated by the thermocouple suggests the possibility that the diminution of film thickness was due more to a decrease in viscosity of the oil than due to a loss of oil. It is noticeable that when the oil was again turned on the film thickness did not rise to its former value until the thermocouple had fallen back to its initial temperature. This supports the suggestion that the changes in film thickness were due to temperature changes, for otherwise, when the oil supply was resumed, the immediate replacement of lost oil is to be expected.

This experimental evidence is, at least, consistent with the suggestion that the oil from the jet does not readily merge with the films already established upon the disks. This suggestion implies that the development of pressure in the oil may not commence where the jet impinges upon the disks; that is at a point remote from the conjunction of the disks on the entry side as is assumed in Martin's theory, but may commence at a point comparatively close to the conjunction. The effect of this would be to reduce the numerical factor in equation (1·1) and, consequently, to reduce the predicted film thickness. This is a possible explanation of the discrepancy, noted at low loads, between the predicted and measured film thicknesses.

5·3. *Results with sliding disks*

A plot of apparent film thickness (h_D^*) against load when the disks were rotating with unequal peripheral speeds (19·6 ft. s^{-1} (597 cm s^{-1}) and 30·2 ft. s^{-1} (920 cm s^{-1}), respectively) is given by curve *d* of figure 8. The important feature of the curve is that it shows a more rapid decrease of film thickness with load when the disks are sliding and rolling than when they are simply rolling.

In another series of experiments the apparent film thickness was measured at frequent intervals after changes in the load. The temperature of the thermocouple was also noted. The results are given in figure 11. The machine was run initially at a load of 1120 Lb. in.$^{-1}$ ($1·97 \times 10^8$ dyn cm^{-1}) until the film thickness was constant. At 6 min the load was decreased to 280 Lb. in.$^{-1}$ ($4·92 \times 10^7$ dyn cm^{-1}) and afterwards capacitance and temperature readings were taken as quickly as possible ($\sim$15 s). These and subsequent readings showed that the film thickness increased with time whilst the temperature decreased. Both again became constant at about 11 min. The change in apparent film thickness at constant load detected in this way was 0·5 μ ($\sim 2 \times 10^{-5}$ in.); approximately one-third of the film thickness. The concurrent change in thermocouple temperature was 5·5° C. The machine was run on at a load of 280 Lb. in.$^{-1}$ up to 100 min, but on three occasions the load was increased to 1120 Lb. in.$^{-1}$ for a sufficient time to take a reading. The load was then reduced to 280 Lb. in.$^{-1}$ again. For convenience all three results are plotted on the 40 min ordinate. At 100 min the load was permanently increased to 1120 Lb. in.$^{-1}$. The film thickness and temperature then moved back close to their starting values.

These results show that some time elapses after the load is changed before the film thickness reaches its new equilibrium value. It should be stated that the film thicknesses plotted in figure 8 were equilibrium values.

5·4. *Discussion*

The friction associated with the sliding component of the relative motion of the two disks will be much greater than that due to the rolling component. Therefore, the change in temperature of the disks with load will be greater when the disks are also sliding than when they are simply rolling. If this temperature affects the film thickness, as already suggested in the discussion of the rolling experiments in which the oil supply was interrupted, the variation of film thickness with load should be greater when sliding is introduced. Curve *d* of figure 8 shows this to be so.

In figure 11 it is shown that following a change of load the film thickness can vary by $0{\cdot}5\,\mu$. This variation occurred at a constant load and, furthermore, the time constants for the changes of film thickness and temperature were the same (figure 11). The change of film thickness must, therefore, be attributed to the change in temperature of the disks. It is even probable that the full change of film thickness with load ($0{\cdot}7\,\mu$) was due to the effect of temperature and not only the $0{\cdot}5\,\mu$ change which was observed after the 15 s delay of the first measurement. It should be noted in this connexion that at the first measurement the thermocouple temperature had already moved by a considerable fraction of its ultimate change.

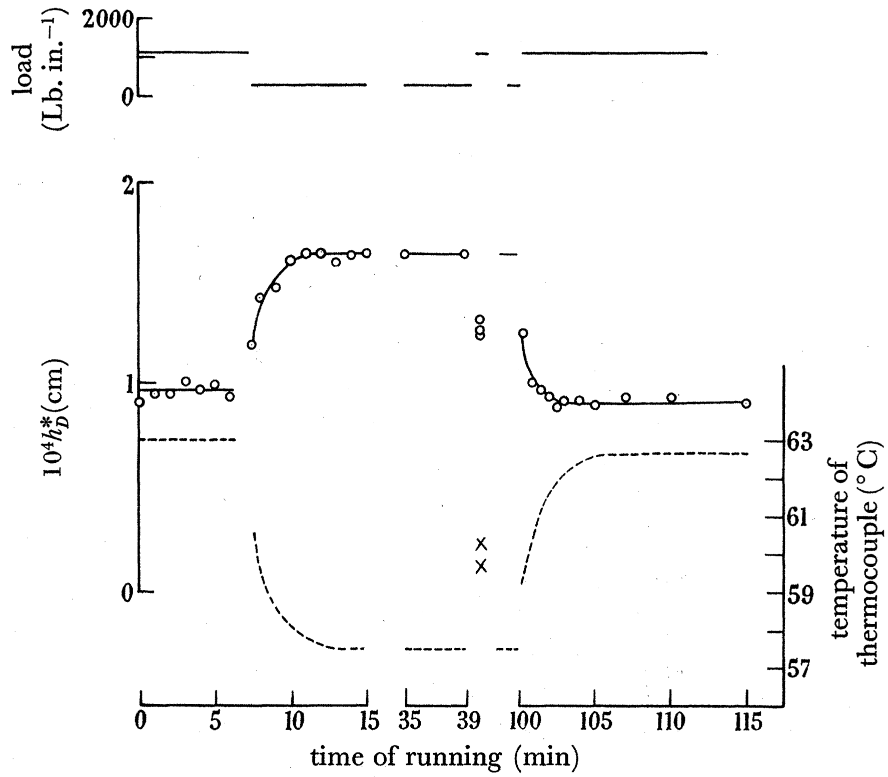

FIGURE 11. The variations of film thickness and disk temperature with time after changing the load upon rolling with sliding disks. (Peripheral speeds 19·6 ft. s^{-1} (597 cm s^{-1}) and 30·2 ft. s^{-1} (920 cm s^{-1}).) —○—, Film thickness; – – –, ×, temperature; —, load.

6. Resistivity and temperature of the oil

It is well known that the capacitance C in e.s.u. and the resistance R in ohms between two electrodes of any geometry immersed in a medium of dielectric constant ϵ and resistivity ρ satisfy the relation

$$CR = \epsilon\rho/4\pi. \tag{6·1}$$

In balancing the bridge both C and R are found so that it only remains to assign a value to $\epsilon/4\pi$ to deduce a mean value for the resistivity of the oil at the conjunction of the disks.

6·1. *Results*

In the upper part of figure 12, values of ρ, calculated from the disk capacitances and resistances given by sliding disks (peripheral speeds 19·6 ft. s^{-1} (597 cm s^{-1}); 30·2 ft. s^{-1} (920 cm s^{-1})) are plotted against load. The equilibrium values of capacitance and resistance were taken and the factor $\epsilon/4\pi$ was assumed to be 0·2. The measured capacitances are plotted beneath the resistivities. The resistances varied from a value beyond the range of the bridge ($>2\times10^7\,\Omega$) at a load of 420 Lb. in.$^{-1}$ down to $9\times10^5\,\Omega$ at a load of 1120 Lb. in.$^{-1}$. It can be seen that ρ varied from a value greater than $2\cdot3\times10^{10}\,\Omega$ cm at a load of 420 Lb. in.$^{-1}$ down to $2\times10^9\,\Omega$ cm at load of 1120 Lb. in.$^{-1}$. Between the same loads the thermocouple temperatures, which are given in the lower curve of the figure, rose from 56 to 63° C. Corresponding results at a load of 1120 Lb. in.$^{-1}$ for conditions of pure rolling show that the resistivity was then greater than $2\cdot3\times10^{10}\,\Omega$ cm, whereas under conditions of rolling with sliding it was only $2\times10^9\,\Omega$ cm.

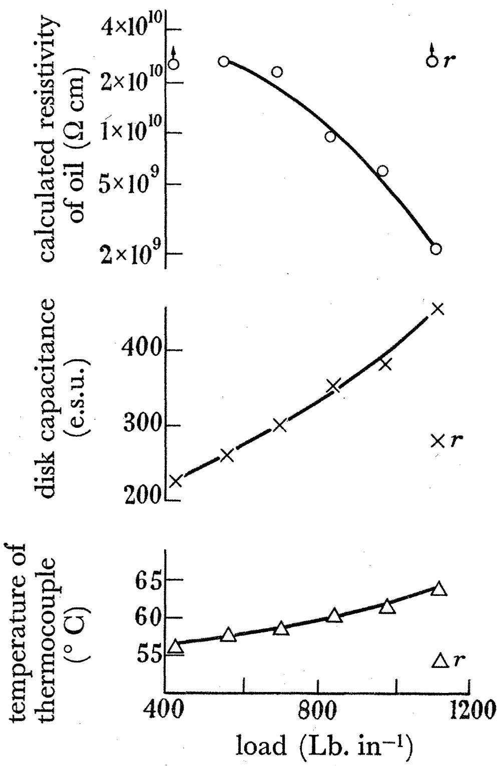

FIGURE 12. Oil resistivity, disk capacitance and disk temperature as functions of load. —○—, resistivity; —×—, capacitance; —△—, temperature. (Peripheral speeds 19·6 ft. s^{-1} (597 cm s^{-1}) and 30·2 ft. s^{-1} (920 cm s^{-1}).) r denotes results with disks rolling at a peripheral speed of 19·6 ft. s^{-1}.)

6·2. *Discussion*

In the calculation of the oil resistivity the factor $\epsilon/4\pi$ was taken as 0·2. This is only approximate for as argued in § 2·2 the dielectric constant will vary with pressure. Moreover, in applying equation (6·1) to the disks only that part of the disk capacitance due to the oil-filled region in which conduction is possible should be admitted. However, the relative values of the calculated resistivities are little affected by these errors, whilst the absolute values will not be put in error by a factor of 10. Here only the order of magnitude of the absolute resistivities is important.

No measurements of oil resisitivity at high pressures are known to the author and the results will be interpreted from resistivities measured at low pressures (Crook 1957). According to these measurements the oil has the resistivity found under sliding conditions at a load of 1120 Lb. in.$^{-1}$, i.e. 2×10^9 Ω cm, at a temperature of 250° C. At high pressures the mobility of the charge carriers will be reduced by the increased viscosity of the oil and, therefore, it is to be expected that an effect of pressure will be to increase the resistivity of the oil. Consequently, the temperature given above should be regarded as a lower limit for the temperature reached by the oil in its passage between the disks.

It has also been shown (figure 12) when the disks are made to roll at the same load of 1120 Lb. in.$^{-1}$ and, therefore, when the oil is subjected to the same pressures, that the resistivity increased by a factor of ten or more to a value exceeding 2×10^{10} Ω cm. This increase of resistivity implies that the temperature of the oil between the disks is less by at least 70° C when the disks are rolling than when they are also sliding at 10 ft. s^{-1}. It can be concluded, therefore, that with sliding the temperature of the oil between the disks (>250° C) is much greater than the surface temperatures of the disks (~65° C) and that when the disks are rolling the oil between them does not attain such a high temperature. The same conclusions also follow from a recollection that the source of heat is at the conjunction of the disks and that the power of the source is far greater when the high friction of sliding has to be overcome than when rolling friction alone opposes motion. This confirms that the resistivity method is qualitatively correct. More significant, however, is the fact that a rise of 5° C in the surface temperatures of the disks at a constant load of 1120 Lb. in.$^{-1}$ (figure 11) is accompanied by as great a decrease of film thickness as that which accompanies the change from pure rolling to rolling with 10 ft. s^{-1} sliding at the same load (curves *b* and *d* of figure 10), although in the latter instance it has now been shown that in addition to the change in the surface temperatures of the disks (figure 12) the temperature of the oil between them rises by 70° C or more when the sliding is introduced. It would seem indeed that the thickness of the oil film is determined more by the surface temperatures of the disks than by the temperature attained by the oil in its passage between them. This implies that the flow of oil between the disks and, therefore, h_D^* are determined by conditions on the entry side ahead of the region in which the viscous forces and heating are intense.

7. Conclusion

It has been shown that when the disks are lightly loaded, the thickness of the hydrodynamic oil film is inversely proportional to the load and proportional to speed as required by Martin's (1916) hydrodynamic theory, but that the absolute film thicknesses are approximately one-half of those to be expected from the theory. It has been shown, as the loads are increased, that the pressure dependence of viscosity begins to affect the film thickness at a load of approximately 15 Lb. in.$^{-1}$ and that at loads exceeding 100 Lb. in.$^{-1}$, the elastic deformation of the disks becomes important. At practical loads (>100 Lb. n.$^{-1}$), the order of magnitude of the film thickness is 1 μ (~4×10^{-5}in.).

At high loads, it has been shown that a change of a few degrees C in the temperature of the disks can affect the thickness of the oil film by as much as 50%. This supports the contention (Crook 1957) that the failure of adequate hydrodynamic lubrication and the consequent damage to the engaging surfaces is largely dependent upon the temperature of these surfaces. From measurements of the resistivity of the oil, estimates have been made of the temperature reached by the oil in its passage between the disks. Temperature rises of 200° C or more have been estimated.

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The experimental results indicate that the theory of the film thickness is complicated. However, the influence of surface temperature upon film thickness and the comparative insensitivity of the film thickness to the temperature reached by the oil, suggests that the key to the problem lies on the entry side, ahead of the region in which the oil becomes heated. This may lead to a considerable simplification of the theory for it gives a primary importance to the least obscure region which has so far received only passing attention in theoretical discussion.

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Appendix. An alternative calibration of the pads

It was shown in § 4·2 that the velocity gradient is constant at the pressure maximum. Therefore if Q_F be the volume rate at which oil is brought up to the pad F (figure 1 b) and h_F^* is the thickness of the film beneath the pad at the pressure maximum,

$$Q_F = \tfrac{1}{2}u_1 h_F^*.$$

If the pads were carrying the maximum load this oil flow could support, the ratio of h_F^* to the minimum pad-disk separation, h_{0F}, would be the same as the similar ratio for the disks, i.e. 1·23 (equation (4·2·4)). If this load were then removed the pad would rise. The value of h_{OF} would increase but h_F^*, since it is determined by the flow, would remain constant. Thus, h_F^*/h_{0F} is less than 1·23 for an unloaded pad and the ratio will be taken as unity. The volume rate of flow of oil beneath the pad is then given by

$$Q_F = \tfrac{1}{2}u_1 h_{0F}.$$

If the inessential but convenient assumption be made that the disks carry films of equal thickness the oil flow between them is given by

$$Q = \tfrac{1}{2}(u_1 + u_2)\, h_{0F}$$

and a comparison of this equation with equation (4·2·3) shows that

$$h_{0F} = h_D^*; \tag{A1}$$

a relationship which is free from assumption concerning conditions at the conjunction of the disks.

The pad capacitance, C_F, depends upon the degree to which the space between the pad and disk is oil filled. If this be known C_F can be calculated in terms of h_{0F} and, therefore, by equation (A1) in terms of the apparent film thickness between the disks.

The disposition of oil beneath the pad was examined by replacing a steel pad by one of glass. Photographs of sufficient contrast for reproduction were not obtained but the salient features are shown in figure 13. An oil-filled band of width l_1 was seen ahead of the centre line, and behind the centre line there was a region of oil and air of width l_2 with the combed formation indicated in figure 13*a*. At the leading edge of the pad a drop of oil developed but calculations and experiments both showed it to have a negligible influence upon C_F. Henceforth it will be ignored.

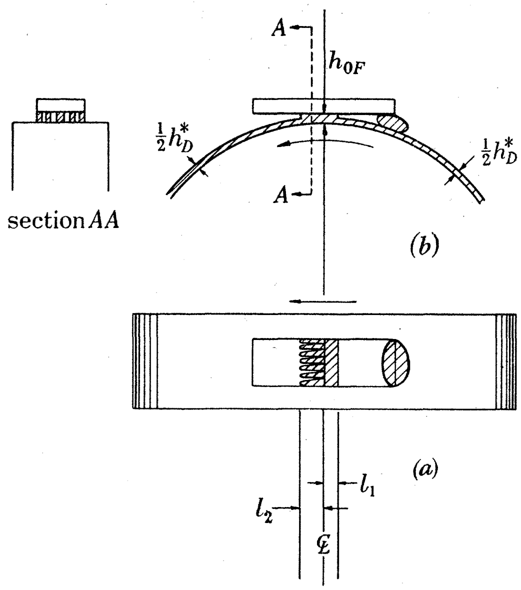

FIGURE 13. The disposition of the oil beneath the pads. (*a*) As seen through a glass pad. (*b*) Elevation.

It will be assumed that the gaps between pad and disk at the outer extremities of the two bands are proportional to h_{0F}. The geometry of pad and disk, which is known because the pads are unloaded, then requires l_1 and l_2 to be proportional to $h_{0F}^{\frac{1}{2}}$. Thus l_2 can be expressed by

$$l_2 = \alpha_2 h_{0F}^{\frac{1}{2}}. \tag{A 2}$$

Calculations of C_F for many assumed dispositions of the oil have all given the result that C_F is proportional to $h_{0F}^{-\frac{1}{2}}$. It will be assumed, therefore, that for this disposition

$$C_F = \beta h_{0F}^{-\frac{1}{2}}. \tag{A 3}$$

From equations (A 2) and (A 3)

$$l_2 C_F = \alpha_2 \beta = \gamma_2, \tag{A 4}$$

i.e. l_2, and similarly l_1, should be inversely proportional to C_F. An experimental confirmation that this is so is given in figure 14. The slopes of the lines give experimental values for γ_1 and γ_2.

The calculation of C_F in terms of h_{0F}, or its equivalent h_D^*, can now be considered. A transverse view of the oil beneath the pad is given in figure 13*b* and attention will be concentrated upon the region with the combed formation; the trailing region of width l_2. In this region the presence of air must ensure that the pressure is everywhere zero. Therefore, $\partial P/\partial x$ is zero and the mean velocity of the oil must be $\frac{1}{2}u_1$. From this mean velocity and the fact that the rate of oil-flow beneath the pad must be everywhere the same the proportion occupied by oil of a section such as AA (figure 13b) may be calculated. When this has been done and after h_D^* has been substituted for h_{0F}, the capacitance per unit face width of this region of oil and air is given in e.s.u. by

$$C = \frac{\sqrt{2r}}{8\pi\sqrt{h_D^*}}\left[(\epsilon+1)\tan^{-1}\phi + \frac{(\epsilon-1)\phi}{(1+\phi^2)}\right],$$

where $$\phi = l_2(2rh_D^*)^{-\frac{1}{2}}.$$

It may be seen from equations (A 1), (A 3) and (A 4) that

$$\phi = \gamma_2/2r\beta.$$

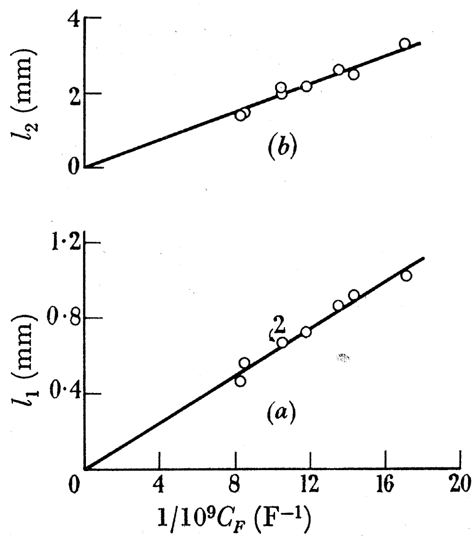

FIGURE 14. The widths of the oil bands beneath the pads as functions of the reciprocal of the pad capacitance. (a) Band of oil ahead of centre line. (b) Band of oil and air behind the centre line.

Similarly, the other components of the total capacitance are found to be functions of γ_1/β or γ_2/β. So C_F may be expressed by

$$C_F = \frac{1}{\sqrt{h_D^*}}[f_1(\gamma_1/\beta) + f_2(\gamma_2/\beta)], \tag{A 5}$$

where γ_1 and γ_2 are known experimentally. Now C_F is also given by equation (A 3) so β must satisfy the condition

$$[f_1(\gamma_1/\beta) + f_2(\gamma_2/\beta)] = \beta.$$

The value of β which satisfies this condition was found and gives

$$C_F = 1{\cdot}67h_D^{*-\frac{1}{2}} \tag{A 6}$$

where C_F is in $\mu\mu$F and h_D^* is in cm.

This derivation of the calibration equation does not rely upon any assumptions concerning the conditions at the conjunction of the disks. Because of the difficulty of knowing precisely the disposition of the oil beneath the pad equation (A 6) is regarded as agreeing satisfactorily with the expression derived by another method in the paper (equation (4·3·1)) and, because the disposition of the oil between the disks is probably less complex, it has been assumed that the latter expression is the more accurate.

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